Lecture 2

Basic Radiometric Quantities. The Beer-Bouguer-Lambert law

Concepts of extinction (scattering plus absorption) and emission.

Objectives:

1. Basic introduction to electromagnetic field:
   - Definitions
   - Dual nature of electromagnetic radiation
   - Electromagnetic spectrum
2. Basic radiometric quantities: energy, intensity, and flux.
3. The Beer-Bouguer-Lambert law. Concepts of extinction (scattering + absorption) and emission. Optical depth.

Required reading:

L80: 1.1

NOTE: There are a few typos in Table 1.1 in Liou's book.

Additional/advanced reading:

Le93: 1.1-1.2 / G&Y: 2.1

NOTE: The various textbooks each might have somewhat different terminology and very different notations.

1. Basic introduction to electromagnetic field.

Electromagnetic radiation is a form of transmitted energy**. *Electromagnetic radiation*** is so-named because it has electric and magnetic fields that simultaneously oscillate in planes mutually perpendicular to each other and to the direction of propagation through space

- Electromagnetic radiation has the **dual nature**:

  its exhibits wave properties and particulate properties.

Wave nature of radiation:

Radiation can be thought of as **a traveling wave.**

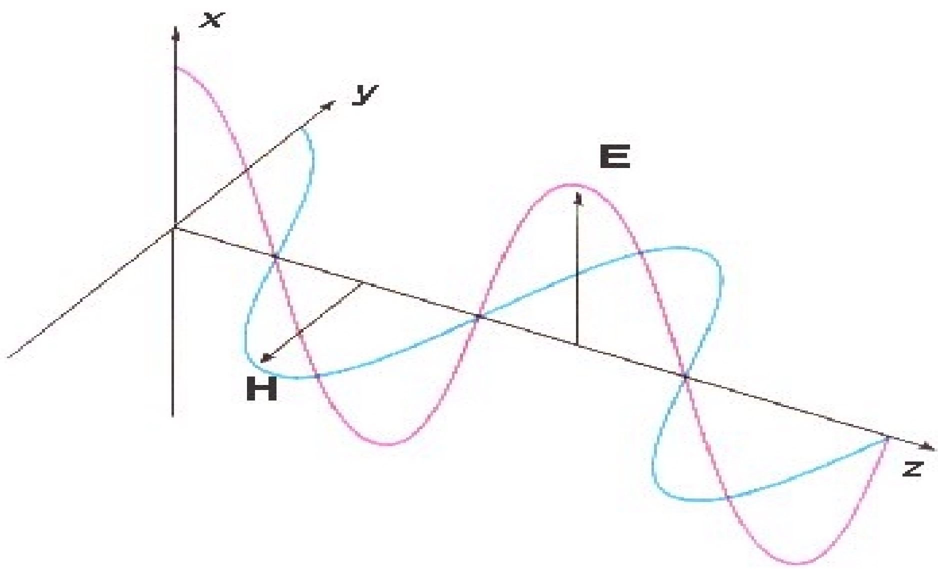


Figure 2.1 A schematic view of an electromagnetic wave propagating along the $\vec{z}$ axis. The electric $\vec{E}$ and magnetic $\vec{H}$ fields oscillate in the x-y plane and perpendicular to the direction of propagation.

Waves are characterized by **wavelength (or frequency)** and **speed**.

- The speed of light in a vacuum: $c = 2.9979 \times 10^8$ m/s $\cong 3.00 \times 10^8$ m/s

Wavelength is the distance between two consecutive peaks or troughs in a wave (symbolized by the Greek letter lambda, λ).

Frequency is defined as the number of waves (*cycles*) per second that pass a given point in space (symbolized by the Greek letter nu, $\tilde{v}$).

Relation between λ and $\tilde{v}$: $\boxed{\lambda\, \tilde{v} = c}$ [2.1]

- Since all types of **electromagnetic radiation** travel at the speed of light, short-wavelength radiation must have a high frequency.

Wavenumber is defined as a count of the number of wave crests (or troughs) in a given unit of length (symbolized by ν):

$$\boxed{\nu = \tilde{\nu}/c = 1/\lambda} \qquad [2.2]$$

UNITS:
Wavelength units: unit length,
Angstrom (A) : 1 A = $1\text{x}10^{-10}$ m;
Nanometer (nm): 1 nm=$1\text{x}10^{-9}$ m;
Micrometer (μm): 1 μm = $1\text{x}10^{-6}$ m;
Frequency units: unit cycles per second 1/s (or s^{-1}) is called hertz (abbreviated Hz)
Wavenumber units: inverse length (often in cm^{-1})

Particulate nature of radiation:

Radiation can be also described in terms of particles of energy, called **photons**.

The energy of a **photon** is given by the expression:

$$\boxed{E_{photon} = h\,\tilde{\nu} = h\,c/\lambda = h\,c\nu} \qquad [2.3],$$

where ***h*** is Plank's constant (h = $6.6256\text{x}10^{-34}$ J s).

- Eq. [2.3] relates energy of each photon of the radiation to the electromagnetic wave characteristics ($\tilde{\nu}$ and λ).

Problem: A light bulb of 100 W emits at 0.5 μm. How many photons are emitted per second?

Solution:

Energy of one photon is E_{photon} = hc/λ, thus, using that 100 W = 100 J/s, the number of photons per second, N, is

$$N(s^{-1}) = \frac{100\,(Js^{-1})\;\lambda(m)}{h(Js)\;c(ms^{-1})} = \frac{100\;\times 0.5\times 10^{-6}}{6.6256\times 10^{-34}\times 2.9979\times 10^{8}} = 2.517\times 10^{20}$$

NOTE: Large number of photons is required because Plank's constant ***h*** is very small!!!

➢ **Spectrum of electromagnetic radiation:**

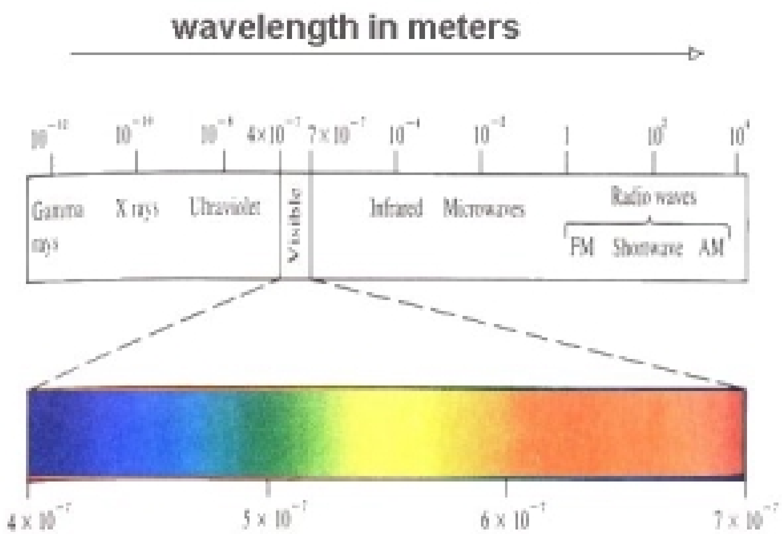


Figure 2.1 The electromagnetic spectrum.

- In this course we study the UV, visible and infrared radiation.

Table 2.1: Relationships between radiation components studied in this course.

Name of spectral region	**Wavelength region, µm**	**Spectral equivalence**
Solar	0.1 - 4	Ultraviolet + Visible + Near infrared = Shortwave
Terrestrial	4 - 100	Far infrared = Longwave
Infrared	0.75 - 100	Near infrared + Far infrared
Ultraviolet	0.1 - 0.38	Near ultraviolet + Far ultraviolet = UV-A + UV-B + UV-C + Far ultraviolet
Shortwave	0.1 - 4	Solar = Near infrared + Visible + Ultraviolet
Longwave	4 - 100	Terrestrial = Far infrared
Visible	0.38 - 0.75	Shortwave - Near infrared - Ultraviolet
Near infrared	0.75 - 4	Solar - Visible - Ultraviolet = Infrared - Far infrared
Far infrared	4 - 100	Terrestrial = Longwave = Infrared - Near infrared
Thermal	4 - 100	Terrestrial = Longwave = Far infrared

Coordinate systems. Solid angle.

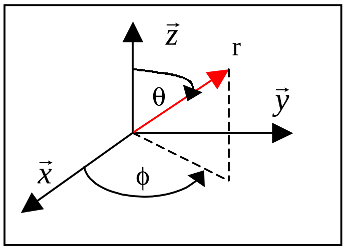
- Both the Cartesian coordinate system and spherical coordinate system are used to characterize the propagation of electromagnetic radiation.

Cartesian (rectangular) coordinate system: three orthogonal unit vectors $\vec{x}, \vec{y}, \vec{z}$

Any vector $\vec{A}$ can be expressed as $\vec{A} = A_x\vec{x} + A_y\vec{y} + A_z\vec{z}$,

and its magnitude is $A = |A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Spherical coordinate system: distance r and the zenithal θ and azimuthal φ angles



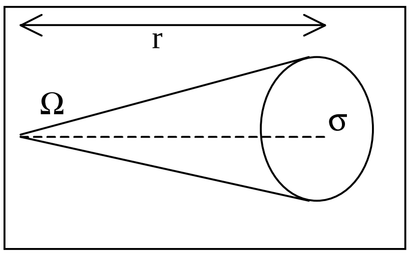
Spherical and rectangular coordinates are related as

x = r sin(θ) cos(φ); y = r sin(θ) sin(φ); z = r cos(θ)

Solid angle is defined as the ratio of the area σ of a spherical surface intercepted by the core to the square of the radius:

$$\Omega = \frac{\sigma}{r^2}$$

UNITS: of a solid angle = steradian (sr)



EXAMPLE: Solid angle of a sphere = 4π

- A differential solid angle can be expressed as

$$d\Omega = \frac{d\sigma}{r^2} = \sin(\theta)\, d\theta\, d\phi \ ,$$

using that a differential area is $d\sigma = (r\, d\theta)\,(r \sin(\theta)\, d\phi)$

2. Basic radiometric quantities.

Intensity (or radiance) is defined as radiant energy in a given direction per unit time per unit wavelength (or frequency) range per unit solid angle per unit area perpendicular to the given direction:

$$I_\lambda = \frac{dE_\lambda}{\cos(\theta) d\Omega dt dA d\lambda} \qquad [2.4]$$

I_λ is referred to as **monochromatic** intensity.

- Monochromatic does not mean at a single wavelengths λ, but in a very narrow (infinitesimal) range of wavelength $\Delta\lambda$ centered at λ.

NOTE: *same name*: intensity = specific intensity = radiance

UNITS: from Eq.[2.4]:
(J sec^{-1} sr^{-1} m^{-2} μm^{-1}) = (W sr^{-1} m^{-2} μm^{-1})

Properties of intensity:

a) In general, intensity is a function of the coordinates ($\vec{r}$), direction ($\vec{\Omega}$), wavelength (or frequency), and time. Thus it depends on seven independent variables: three in space, two in angle, one in wavelength (or frequency) and one in time.
b) Intensity, as a function of position and direction, gives a complete description of the electromagnetic field.

- If intensity does not depend on the direction, the electromagnetic field is said to be **isotropic.** If intensity does not depend on position the field is said to be **homogeneous**.

Flux (or irradiance) is defined as radiant energy in a given direction per unit time per unit wavelength (or frequency) range per unit area perpendicular to the given direction:

$$F_{\lambda} = \frac{dE_{\lambda}}{dt dA d\lambda} \qquad [2.5]$$

NOTE: L80 refers to this quantity as “flux density”. We will be using the term “flux”.

From Eqs. [2.4]-[2.5]:

$$F_{\lambda} = \int_{\Omega} I_{\lambda} \cos(\theta) d\Omega \qquad [2.6]$$

Thus monochromatic **flux** is the integration of normal component of monochromatic **intensity** over the all solid angles over the hemisphere.
Eq. [2.6] in spherical coordinates:

$$F_{\lambda} = \int_{0}^{2\pi} \int_{0}^{\pi/2} I_{\lambda}(\theta, \varphi) \cos(\theta) \sin(\theta) d\varphi d\theta$$

NOTE: For isotropic radiation $F_{\lambda} = \pi I_{\lambda}$

- **Integral** (or total) **intensity** ***I*** and **flux** *F* are determined by integrating over the wavelength the monochromatic intensity and flux, respectively:

$$I = \int_{0}^{\infty} I_{\lambda} d\lambda \qquad\qquad F = \int_{0}^{\infty} F_{\lambda} d\lambda$$

NOTE: Concepts used to describe the scattering processes of electromagnetic radiation (such as Stokes matrix and polarization) will be defined in Lecture 13.

3. The Beer-Bouguer-Lambert law. Concepts of extinction (scattering + absorption) and emission.

Electromagnetic radiation in the atmosphere interacts with gases, aerosol particles, and cloud particles.

- **Extinction** and **emission** are two main types of the interactions between an electromagnetic radiation field and a medium (e.g., the atmosphere).

General definition:

Extinction is a process that decreases the radiant **intensity**, while **emission** increases it.

NOTE: "same name": **extinction = attenuation**

Radiation is **emitted** by **all** bodies that have a temperature above absolute zero (O K) (often referred to as **thermal emission**).

- **Extinction** is due to **absorption** and **scattering**.

Absorption is a process that removes the radiant energy from an electromagnetic field and transfers it to other forms of energy.

Scattering is a process that **does not** remove energy from the radiation field, but may redirect it.

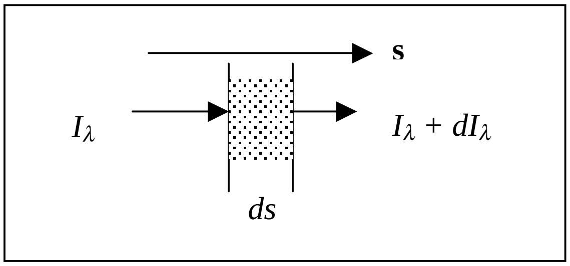
NOTE: Scattering can be thought of as **absorption** of radiant energy followed by **re-emission** back to the electromagnetic field with negligible conversion of energy. Thus, scattering can remove radiant energy of a light beam traveling in one direction, but can be a "source" of radiant energy for the light beams traveling in other directions.

NOTE: Scattering processes will be discussed in details starting with Lecture 13.

The fundamental law of extinction is the **Beer-Bouguer-Lambert law,** which states that the extinction process is linear in the intensity of radiation and amount of matter, provided that the physical state (i.e., T, P, composition) is held constant.

NOTE: Some non-linear processes do occur as will be discussed later in the course.

Consider a small volume ***ΔV*** of infinitesimal length ***ds*** and area ***ΔA*** containing optically active matter. Thus, the change of intensity along a path ***ds*** is proportional to the amount of matter in the path.



For extinction: $dI_\lambda = -\beta_{e,\lambda} I_\lambda ds$

For emission: $dI_\lambda = \beta_{e,\lambda} J_\lambda ds$

where $\beta_{e,\lambda}$ is the **volume extinction coefficient** (LENGTH^{-1}) and J_λ is the **source function**.

- In the most general case, the **source function** J_λ has emission and scattering contributions.

NOTE: Volume extinction coefficient is often referred to as the **extinction coefficient.**

- Generally, the **volume extinction coefficient** is a function of position **s.**
  (Sometimes it may be expressed mathematically as $\beta_{e,\lambda}$ (s), but s is often dropped).

Extinction coefficient = absorption coefficient + scattering coefficient

$$\boxed{\beta_{e,\lambda} = \beta_{a,\lambda} + \beta_{s,\lambda}}$$

- **Extinction coefficient (as well as absorption and scattering coefficients)** can be expressed in different forms according to the definition of the amount of matter (e.g., number concentrations, mass concentration, etc.) of matter in the path (see Lecture 4).

- **Volume** and **mass extinction coefficients** are most often used.

Mass extinction coefficient = volume extinction coefficient/density

UNITS: the mass coefficient is in unit area per unit mass ($LENGTH^2$ $MASS^{-1}$). For instance: (cm^2 g^{-1}), (m^2 kg^{-1}), etc.

If ρ is the density (mass concentration) of a given type of particles (or molecules), then

$$\beta_{e,\lambda} = \rho\beta^*_{e,\lambda}$$

$$\beta_{s,\lambda} = \rho\beta^*_{s,\lambda}$$

$$\beta_{a,\lambda} = \rho k_\lambda$$

where the $\boldsymbol{\beta^*_{e,\lambda}}$; $\boldsymbol{\beta^*_{s,\lambda}}$, and $\boldsymbol{k_\lambda}$ are the **mass extinction, scattering, and absorption coefficients**, respectively.

NOTE: These notations differ from those in L80. Note that L80 uses $\boldsymbol{k_\lambda}$ for the both mass extinction coefficient and mass absorption coefficient!!!

- The **extinction cross section** of a given particle (or molecule) is a parameter that measures the attenuation of electromagnetic radiation by this particle (or molecule).

In the same fashion, **scattering and absorption cross sections** can be defined.

UNITS: the cross section is in unit area ($LENGTH^2$)

If N is the particle (or molecule) number concentration of a given type of particles (or molecules), then

$$\beta_{e,\lambda} = \sigma_{e,\lambda} N$$

$$\beta_{s,\lambda} = \sigma_{s,\lambda} N$$

$$\beta_{a,\lambda} = \sigma_{a,\lambda} N$$

where $\sigma_{e,\lambda}$, $\sigma_{s,\lambda}$, and $\sigma_{a,\lambda}$ are the extinction, scattering, and absorbing cross sections, respectively.

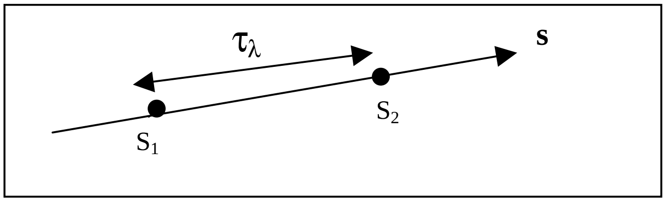
UNITS: Particle number concentration is in the number of particles per unit volume (LENGTH^{-3}).

NOTE: IR absorption by gases will be discussed in details in Lecture 5-6

Solar absorption by gases will be discussed in details in Lecture 11. Absorption by particulate matter (e.g., aerosol and cloud particles) will be discussed in Lecture 8. Scattering will be discussed in details staring with Lecture 13.

- **Optical depth** of a medium between points s_1 and s_2 is defined as

$$\tau_\lambda(s_2; s_1) = \int_{s_1}^{s_2} \beta_{e,\lambda}(s) ds$$



UNITS: optical depth is unitless.

NOTE: "same name": **optical depth = optical thickness = optical path**

- **If** $\beta_{e,\lambda}$ (s) does not depend on position (called a homogeneous optical path), thus

$\beta_{e,\lambda}(s) = <\beta_{e,\lambda}>$ and $\tau_\lambda(s_2; s_1) = <\beta_{e,\lambda}>(s_2 - s_1) = <\beta_{e,\lambda}> s$

For this case, the **Extiction law** can be expressed as

$$\boxed{I_\lambda = I_0 \exp(-\tau) = I_0 \exp(-<\beta_{e,\lambda}> s)} \quad [2.7]$$

Optical depth can be expressed in several ways:

$$\boxed{\tau_\lambda(s_1;s_2) = \int_{s_1}^{s_2} \beta_{e,\lambda} ds = \int_{s_1}^{s_2} \rho \beta^*_{e,\lambda} ds = \int_{s_1}^{s_2} N \sigma_{e,\lambda} ds} \qquad [2.8]$$

- If in a given volume there are several types of optically active particles each with $\boldsymbol{\beta^i_{e,\lambda}}$, etc., then the optical depth can be expressed as:

$$\tau_\lambda = \sum_i \int_{s_1}^{s_2} \beta^i_{e,\lambda} \, ds = \sum_i \int_{s_1}^{s_2} \rho_i \beta^{*i}_{e,\lambda} ds = \sum_i \int_{s_1}^{s_2} N_i \sigma^i_{e,\lambda} ds \qquad [2.9]$$

where $\boldsymbol{\rho_i}$ and $\boldsymbol{N_i}$ is the mass concentrations (densities) and particles concentrations of the ***i***-th species.